$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$-|\theta| \leqslant \sin\theta \leqslant |\theta|$ and $-|\theta| \leqslant 1-\cos\theta \leqslant |\theta|$

limit $\epsilon - \delta$ definition

$\lim_{x\to a} f(x) = L.$ $\forall \epsilon > 0.$ $\exists \delta > 0.$ s.t if $0 < |x-a| < \delta$, then $0 < |f(x)-L| < \epsilon$.

One-side limits formal definition.

Let $f: D \subseteq \mathbb{R} \to \mathbb{R}$

① Right-hand side

$\lim_{x\to c^+} f(x) = L$ if for every $\epsilon > 0$. there exists $\delta > 0$. s.t if $x \in D$ and $c < x < c+\delta$, then $|f(x)-L| < \epsilon$

② Left-hand side

$\lim_{x\to c^-} f(x) = L$ if for every $\epsilon > 0$. there exists $\delta > 0$. s.t if $x \in D$ and $c-\delta < x < c$ then. $|f(x)-L| < \epsilon$

Continuity.

A function is continuous at a point $x=a$. if all three of the following conditions are satisfied:

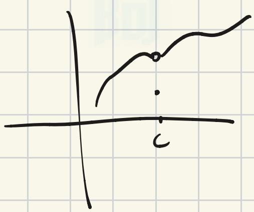
1. $f(a)$ is defined.

2. $\lim_{x\to a} f(x)$ exists

3. $\lim_{x\to a} f(x) = f(a)$

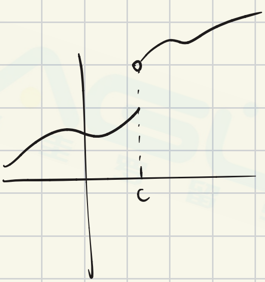
Discontinuity.

1. Removable discontinuity.



$\lim_{x\to c} f(x) \neq f(c)$.

2. Jump discontinuity.



$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$

they both exist.

3. Infinite discontinuity:

$\lim_{x\to c} f(x)$ not exists

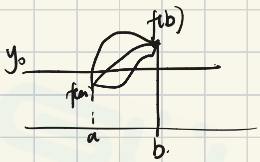
4. Oscillatory discontinuity

$\lim_{x\to c} f(x)$ not exist

Intermediate Value Theorem

If a function f is continuous on a closed interval $[a,b]$, and y_0 is any number between $f(a)$ and $f(b)$, then there exists at least one number c in (a,b) s.t

$f(c) = y_0$.



Finite limits at infinity.

$\lim_{x \to \infty} f(x) = L$ $\quad \forall \epsilon > 0. \quad \exists M$ st if $x > M$. then $|f(x) - L| < \epsilon$.

For example.

$\lim_{x \to \infty} \frac{1}{x} = 0$ $\quad \forall \epsilon > 0. \ \exists M$. st if $x > M$. then $|\frac{1}{x} - 0| < \epsilon$.

$|\frac{1}{x}| < \epsilon$. $\quad \frac{1}{x} < \epsilon \quad x > \frac{1}{\epsilon}$ $\quad$ then we choose $M = \frac{1}{\epsilon}$.

So let $\epsilon > 0$. $\quad M = \frac{1}{\epsilon}$.

when $x > M$. $\Rightarrow x > \frac{1}{\epsilon} \Rightarrow |\frac{1}{x} - 0| < \epsilon$.