L1-2

Point-Direction line : $l(t) = \vec{a} + t\vec{v}$ $\vec{a}$ — pass through $\vec{v}$ — direction

Coordinate form $\begin{cases} x = x_1 + at \\ y = y_1 + bt \\ z = z_1 + ct \end{cases}$

Relation between lines : ① Parallel ② Intersecting ③ Skew : if not ① or ②

Norm : $\|\vec{v}\| = \sqrt{v_1^2 + \cdots + v_n^2}$ Distance : $d(P,Q) = \sqrt{\Sigma(y_i - x_i)^2}$ Unit vector : $\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$

Dot product : $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$ $\mathrm{Proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\cdot\vec{v}$ $\vec{u}$ on $\vec{v}$ $\vec{u}\cdot\vec{v} = \|\vec{v}\|\cdot\|\mathrm{proj}_{\vec{v}}(\vec{u})\|$

$\|\vec{v} - \vec{u}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\vec{u}\cdot\vec{v}$

Cauchy-Schwarz inequality : $|\vec{u}\cdot\vec{v}| \le \|\vec{u}\|\|\vec{v}\|$ $\vec{u}$和$\vec{v}$平行时取等 $\Rightarrow$ $\|\vec{u}+\vec{v}\| \le \|\vec{u}\| + \|\vec{v}\|$ 三角不等式

Cross product : $\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\|\sin\theta$ $\hat{i}$ $\hat{k} \leftarrow \hat{j}$ Area of Δ : $\frac{1}{2}\|a\times b\|$ 平四 : $\|a\times b\|$

L3-4

Plane in 3D : $ax + by + cz = d$ Normal vector : $\vec{n} = (a,b,c)$ $\vec{n}\cdot((x,y,z) - P_0) = 0$

Distance from point to plane : $\mathrm{dist}(Q,\Pi) = \frac{|ax_Q + by_Q + cz_Q - d|}{\sqrt{a^2+b^2+c^2}}$

Parametric plane : $\vec{r}(s,t) = \vec{r_0} + s\vec{u} + t\vec{v}$ $s,t \in \mathbb{R}$ $\vec{n} = \vec{u}\times\vec{v}$

Angle between two planes : $\cos\alpha = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\|\vec{n_2}\|}$ $\alpha \in [0, \frac{\pi}{2}]$

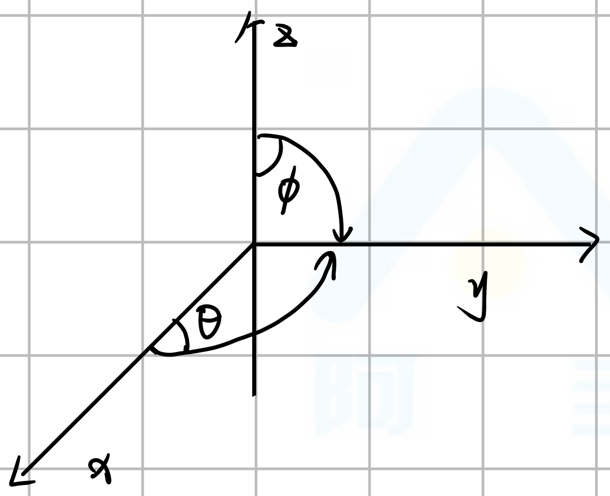
Line in 3D parametric : $\vec{r}(t) = \vec{r_0} + t\vec{u}$ $\begin{cases} x = x_0 + v_x t \\ y = y_0 + v_y t \\ z = z_0 + v_z t \end{cases}$ $\Rightarrow$ $\frac{x - x_0}{v} = \frac{y - y_0}{v} = \frac{z - z_0}{v}$

交线：两平面法向量相乘为线方向向量，找一交点即可

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \arctan(\frac{y}{x}) \end{cases}$ If centered at (x_0, y_0) $\begin{cases} x = x_0 + r\cos\theta \\ y = y_0 + r\sin\theta \end{cases} \Rightarrow \begin{cases} r = \sqrt{(x-x_0)^2+(y-y_0)^2} \\ \theta = \arctan(\frac{y-y_0}{x-x_0}) \end{cases}$

Cylindrical to Cartesian $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$ Back $\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \operatorname{atan2}(y, x) \\ z = z \end{cases}$ $\theta \in [0, 2\pi]$

Spherical to Cartesian $\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$ Back $\begin{cases} \rho = \sqrt{x^2+y^2+z^2} \\ \theta = \operatorname{atan2}(y, x) \\ \phi = \arccos(z/\rho) = \operatorname{atan2}(\sqrt{x^2+y^2}, z) \end{cases}$ $\phi \in [0, \pi]$



$$\operatorname{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}), & x > 0 \\ \arctan(\frac{y}{x}) + \pi, & x < 0, y \geq 0 \\ \arctan(\frac{y}{x}) - \pi, & x < 0, y < 0 \\ \frac{\pi}{2}, & x = 0, y > 0 \\ -\frac{\pi}{2}, & x = 0, y < 0 \\ \text{undefined}, & x = 0, y = 0 \end{cases}$$

Elliptic to Cartesian $\begin{cases} x = c\cosh\mu\cos\nu \\ y = c\sinh\mu\sin\nu \end{cases}$ 焦点 $(c, 0)$ $(-c, 0)$ (μ, ν)

$a = c\cosh\mu_0$ $b = c\sinh\mu_0$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $c = \sqrt{a^2 - b^2}$

Topology → 研究集合的形状性质

Open set： $U \subset \mathbb{R}^n$ 是开集，if for any $x \in U$, exist $\varepsilon > 0$ s.t. $B_\varepsilon(x) \subset U$

$B_\varepsilon(x) = \{y \in \mathbb{R}^n : \|y - x\| < \varepsilon\}$ called a open ball centered at x, radius with ε.

集合里面的每一个点的周围，都能画一个小球，而且这个小球完全待在集合里面，即为开集。 $(0, 1)$

Closed set : $F \subset \mathbb{R}^n$ 是闭集，if $\mathbb{R}^n / F$ is open → F closed ⟺ F^c open　　$[0,1]$

Open ball : $B_r(x_0) = \{x \in \mathbb{R}^n : \|x - x_0\| < r\}$

Closed ball : $\bar{B}_r(x_0) = \{x \in \mathbb{R}^n : \|x - x_0\| \le r\}$

Boundary : ∂A → 在其周围画任何小圆，都会同时碰到集合里面和集合外面

$\partial A = \bar{A} \setminus \mathrm{int}(A)$　　$\partial A = \bar{A} \cap \bar{A^c}$　　$\bar{A}$ → Closure　　$\mathrm{int}(A)$ → Interior

Interior point : If $x \in A$, exist $\varepsilon > 0$, s.t. : $B_\varepsilon(x) \subset A$　then x called the interior point of A.

这个点周围能画一个小球，完全待在集合里面

Interior : $\mathrm{int}(A) = \{x \in A : \exists \varepsilon > 0, B_\varepsilon(x) \subset A\}$

Open set and interior : U is open ⟺ $U = \mathrm{int}(U)$

Accumulation point : If $(B_\varepsilon(x) \setminus \{x\}) \cap A \neq \emptyset$

在 x 附近画任何小球，都能找到别的A中的点　可以不属于A

无限小的

Closure : $\bar{A} = A \cup A'$　　A' → Accumulation point 的集合

$\bar{A}$ = 集合A加上它所有应该补上的边界点

Connect sets : $(0,1)$ ✓　　$(0,1) \cup (2,3)$ ✗　　Path-connected ⇒ Connected

Domain : $(0,1)$ ✓　$(0,1) \cup (2,3)$ ✗　$[0,1]$ ✗　nonempty + connect + open set

Bounded : Ω is bounded if exist $x_0 \in \mathbb{R}^n$ and $R > 0$ s.t. $\Omega \subset B_R(x_0)$

$(0,1)$ 有界，可以被 $(-10,10)$ 包住；$\mathbb{R}$ 无界，无法包住整个实数轴

Compact sets : K compact ⟺ K closed and bounded　闭且有界