Lecture7_CN

第七讲：非齐次线性常微分方程——参数变易法与待定系数法

Variation of parameters

In general, we will have to find the solution to the system of equations

$$\begin{cases} u_1'(t)y_1(t) + u_2'(t)y_2(t) &= 0 \\ u_1'(t)y_1'(t) + u_2'(t)y_2'(t) &= g(t), \end{cases}$$

which can be written in its matricial form.

Then we have by Cramer's rule that

$$u_1'(t) = \frac{-y_2(t)g(t)}{W[y_1, y_2](t)}, \qquad u_2'(t) = \frac{y_1(t)g(t)}{W[y_1, y_2](t)}.$$

Recall: $W[y_1, y_2](t) = y_1(t)y_2'(t) - y_2(t)y_1'(t)$.

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总结

概念	关键公式 / 方法
通解结构	$y = y_h + y_p$，其中 y_h 是齐次解，y_p 是特解
参数变易法	$y_p = -y_1 \int \frac{y_2 \cdot g(t)}{W} dt + y_2 \int \frac{y_1 \cdot g(t)}{W} dt$
朗斯基行列式	$W[y_1, y_2](t) = y_1(t)y_2'(t) - y_1'(t)y_2(t)$
待定系数法	根据 $g(t)$ 的形式猜测 y_p，求解系数
修正规则	若猜测与 y_h 重叠，乘以 t（或 t^k，k 为重数）